The STEM Math Boot Camp Challenge

Here are a dozen challenging math problems that any STEM student should be able to solve quickly and easily if you know how to use the proper tool. If you can complete this quiz in less than an hour, you should be ready to compete with the best educated peers you will have in a good STEM University.

If not, then you will benefit greatly from joining the STEM Math Boot Camp.

1. **Let $P(x) = x^5 + 4.1x^4 + 1.1x^3 - 8.2x^2 - 27.4x - 37.5$**
   Find the Roots of P(x), both real and complex, to three significant digits. [This is a Pre-calculus problem that is very difficult without a modern tool.]

2. **Find the relative Maximum and Minimum Points of P(x), and find the intervals where P(x) is increasing and decreasing.**
   [This is a Differential Calculus Problem.]

3. **Find the points of inflection of P(x) and the Concavity intervals and graph P(x). [This is a Differential Calculus Problem.]**

4. **Let $F(x) = x^2 + .5Sin(10x)$. Graph F(x) from x = 1 to 3, Find the Arc Length of this graph, and find the area beneath the graph from 1 to 3. [This is an Integral Calculus Problem.]**

5. **Rotate F(x) from problem #4 about the x-axis and find both the Volume and Surface Area of the Solid of Revolution. [This is an Integral Calculus Problem.]**

6. **Let $G(x) = Sin(x^2)$. Find the anti-derivative of G(x) and the Area under the graph of G(x) from x = -1.5 to 1.5 [This is an Integral Calculus Problem that would probably not be given in a classical calculus course since there**

is not an anti-derivative of G(x) consisting of the standard functions, and thus one could not easily apply the Fundamental Theorem of Calculus.]

7. Find the point of intersection of the three planes:
   $3.1x + 4.3y - 7.6z = 5.2$, $2.7x - 3.4y + 5.1z = -6.9$, $-0.9x + 4.2y - 3.8z = 8.7$
   [This is a Linear Algebra problem you should be able to solve in about two minutes with a modern tool.]

8. Let $F(x) = 1.9/(3.1 + 2.7x^2)$, Find a polynomial, P(x), of degree 8 which is the best approximation of F(x) at x = 0. Plot both F(x) and P(x) and observe that P(x) is a very good approximation of F(x) from $-.5<x<.5$
   [This is an advanced Differential Calculus Problem that would be very time consuming classically, but very easy with a modern tool.]

9. y is a function of x. Find the solution to the differential equation:
   $y'' + y' + y = \cos(x)$, with initial conditions $y(0) = .5$, and $y'(0) = .3$
   [This is a Differential Equation of order 2.]

10. y is a function of t. Find the solution to the differential equation:
   $y' + y + e^t = 0$, $y(1) = 2$ [This is a Differential Equation of order 1.]

11. $x^2 + y^3 - (xy)^2 = 0$ implicitly defines three functions y of x. What is the asymptotic behavior of these three functions as $x \to \infty$ and $x \to -\infty$?
   Also, graph these three functions and their asymptotes.

12. What is the length of the curve defined parametrically (sin(t),cos(3t)) from t = 0 to Pi? Graph the curve.

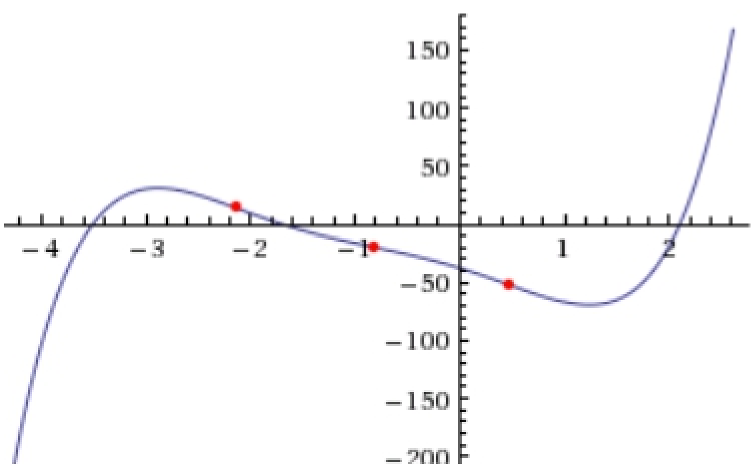
Answers:

1. **-3.51, -1.700, +2.10, -0.498 + 1.66i, -0.498 - 1.66i**

2. **(-2.89, 31.06) is a Maximum, (1.24, -69.4) is a Minimum. P(x) is increasing in the intervals (-∞, -2.89) and (1.24, +∞), and decreasing in the interval (-2.89, 1.24)**

3. **(-2.12, 13.3) and (-0.813, -19.8) and (0.475, -52.02) are the three inflection points and P(x) is concave down in the intervals (-∞, -2.12) and (-0.813, 0.475) and concave up in the intervals (-2.12, -0.813) and (0.475, ∞)**

   The Graph is below and the points of inflection are given on it.



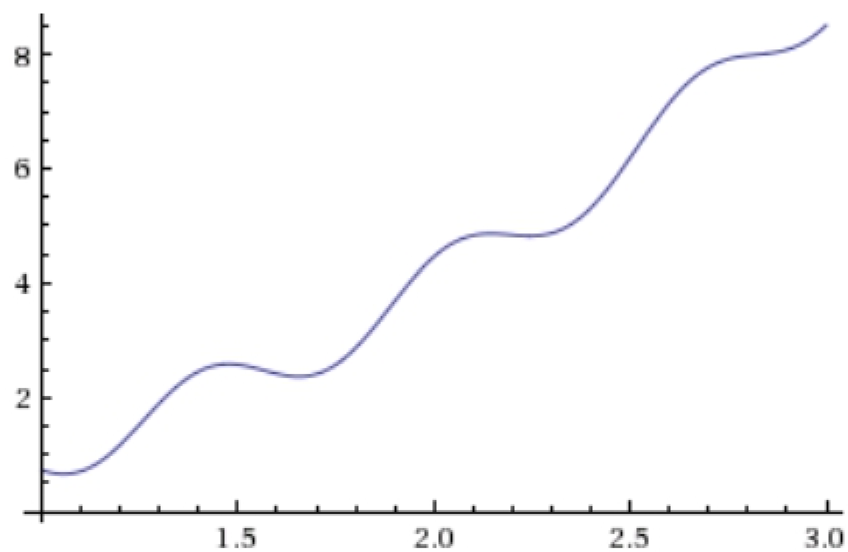
4. **The Arc Length of the Graph of F(x) from 1 to 3 is:**

$$\int_1^3 \sqrt{1 + (2x + 5.\cos(10x))^2}\, dx \approx 8.9013!$$

The Area under the Graph of F(x) from x = 1 to 3 is:

$$\int_1^3 \left(x^2 + 0.5 \sin(10x)\right) dx = 8.617$$

Here is the Graph of F(x)



5. **Volume of Solid of Revolution of F(x) about the x axis:**

$$\int_1^3 \pi \left(x^2 + 0.5 \sin(10x)\right)^2 dx = 152.017$$

Surface Area of Solid of Revolution of F(x) about the x axis:

$$\int_1^3 2\pi \left|x^2 + 0.5 \sin(10x)\right| \sqrt{1 + (2x + 5.\cos(10x))^2}\, dx = 247.897$$

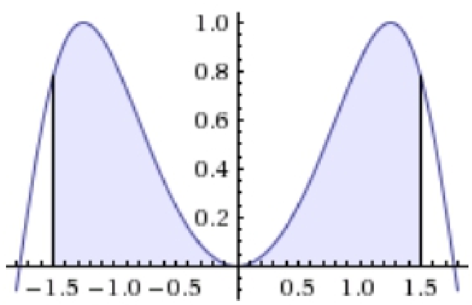
6. **The anti-derivative of Sin(x^2) is a Special Function called a Fresnal Integral:**

$$\int \sin(x^2)\,dx = \sqrt{\frac{\pi}{2}}\, S\!\left(\sqrt{\frac{2}{\pi}}\, x\right) + \text{constant}$$

The Area under the graph of Sin(x^2) from -1.5 to 1.5 is:

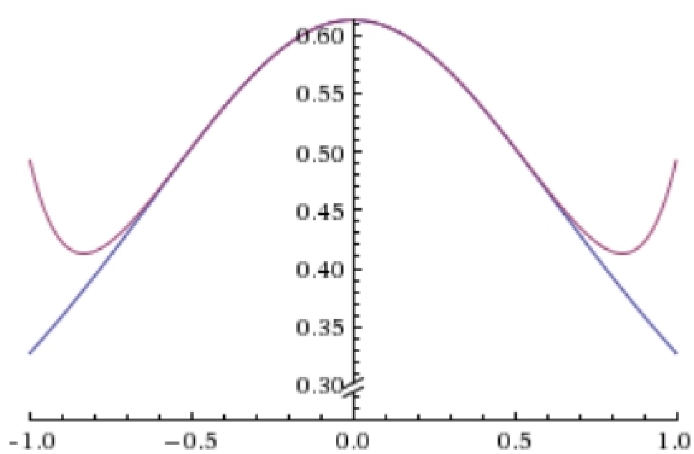
$$\int_{-1.5}^{1.5} \sin(x^2)\,dx = 1.55648$$

Visual representation of the integral:



7. **x = -0.451, y = 2.44, z =.510**

8. **P(x) = $0.353x^8 - 0.405x^6 + 0.465x^4 - 0.534x^2 + 0.613$**



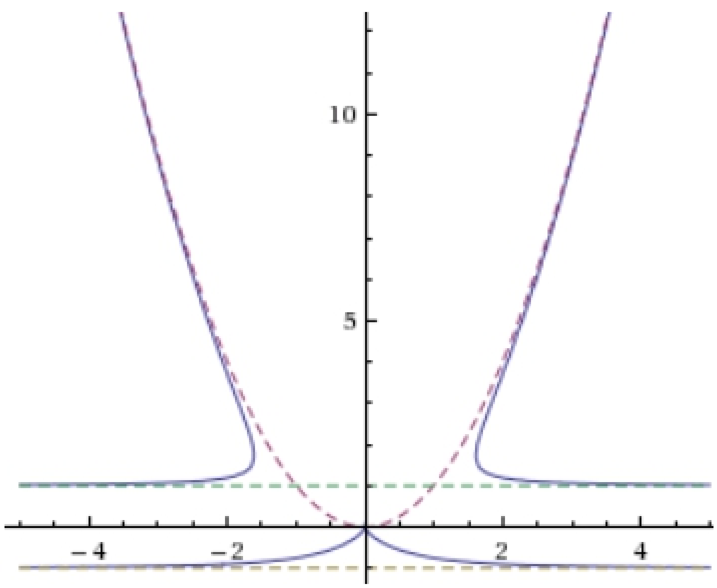
9. The solution of the differential equation is:

$$y(x) = \sin(x) - 0.519615\, e^{-x/2} \sin\left(\frac{\sqrt{3}\, x}{2}\right) + 0.5\, e^{-x/2} \cos\left(\frac{\sqrt{3}\, x}{2}\right)$$

10. The solution of the differential equation is:

$$y(t) = \frac{1}{2}\, e^{-t} \left(-e^{2t} + 4\, e + e^{2}\right)$$

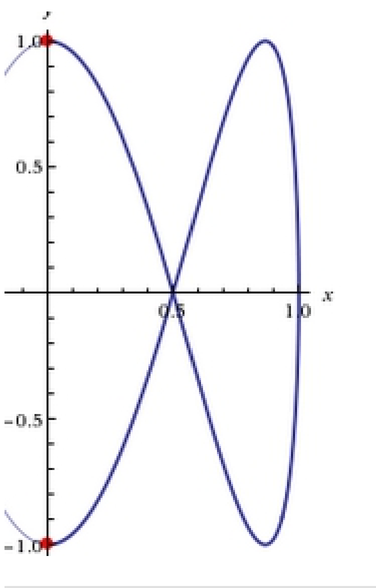
11. One function approaches y = x^2 asymtotically in both directions, and a second function approaches y = +1 in both directions, and the third function approaches y = -1 in both directions. Notice the symmetry.



12. The length is:

$$\int_0^{\pi} \sqrt{\cos^2(t) + 9\sin^2(3t)}\ dt \approx 6.5327$$

The Graph is:



The tools you will learn to use as a STEM professional will solve much more difficult math problems very easily. These modern tools will, in fact, enable the STEM professional to solve problems that would be simply intractable without the modern tools.

It is IMPERATIVE that a 21st Century STEM student learns and masters the modern tools of STEM.

That is the Mission of Triad Math's STEM Math Boot Camp.

For more information, please visit www.STEMMathMadeEasy.com